CHAPTER 1

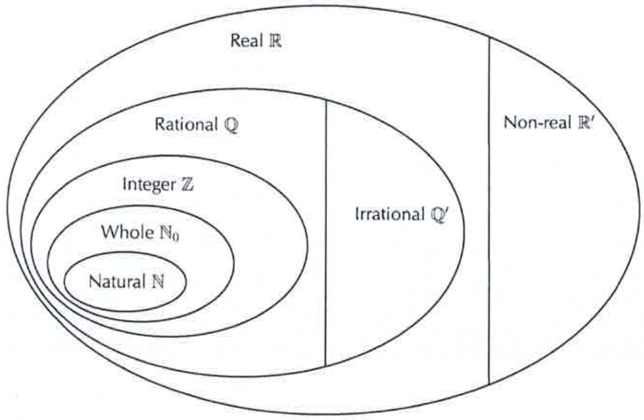
Exponents and surds

1.1	*Revision*	4
1.2	*Rational exponents and surds*	8
1.3	*Solving surd equations*	19
1.4	*Applications of exponentials*	23
1.5	*Summary*	25

1.1 Revision EMBF2

The number system EMBF3

The diagram below shows the structure of the number system:



See video: 2222 at www.everythingmaths.co.za

We use the following definitions:

- $\mathbb{N}$: natural numbers are $\{1;\ 2;\ 3;\ \ldots\}$
- $\mathbb{N}_0$: whole numbers are $\{0;\ 1;\ 2;\ 3;\ \ldots\}$
- $\mathbb{Z}$: integers are $\{\ldots;\ -3;\ -2;\ -1;\ 0;\ 1;\ 2;\ 3;\ \ldots\}$
- $\mathbb{Q}$: rational numbers are numbers which can be written as $\frac{a}{b}$ where a and b are integers and $b \neq 0$, or as a terminating or recurring decimal number.
  Examples: $-\frac{7}{2};\ -2{,}25;\ 0;\ \sqrt{9};\ 0{,}\dot{8};\ \frac{23}{1}$
- $\mathbb{Q}'$: irrational numbers are numbers that cannot be written as a fraction with the numerator and denominator as integers. Irrational numbers also include decimal numbers that neither terminate nor recur.
  Examples: $\sqrt{3};\ \sqrt[5]{2};\ \pi;\ \frac{1+\sqrt{5}}{2};\ 1{,}27548\ldots$
- $\mathbb{R}$: real numbers include all rational and irrational numbers.
- $\mathbb{R}'$: non-real numbers or imaginary numbers are numbers that are not real.
  Examples: $\sqrt{-25};\ \sqrt[4]{-1};\ -\sqrt{-\frac{1}{16}}$

See video: 2223 at www.everythingmaths.co.za

Exercise 1 – 1: The number system

Use the list of words below to describe each of the following numbers (in some cases multiple words will be applicable):

- Natural ($\mathbb{N}$)
- Whole ($\mathbb{N}_0$)
- Integer ($\mathbb{Z}$)
- Rational ($\mathbb{Q}$)
- Irrational ($\mathbb{Q}'$)
- Real ($\mathbb{R}$)
- Non-real ($\mathbb{R}'$)

1. $\sqrt{7}$
2. 0,01
3. $16\frac{2}{5}$
4. $\sqrt{6\frac{1}{4}}$
5. 0
6. 2π
7. $-5{,}3\dot{8}$
8. $\frac{1-\sqrt{2}}{2}$
9. $-\sqrt{-3}$
10. $(\pi)^2$
11. $-\frac{9}{11}$
12. $\sqrt[3]{-8}$
13. $\frac{22}{7}$
14. 2,45897...
15. $0{,}\overline{65}$
16. $\sqrt[5]{-32}$

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 2224	2. 2225	3. 2226	4. 2227	5. 2228	6. 2229
7. 222B	8. 222C	9. 222D	10. 222F	11. 222G	12. 222H
13. 222J	14. 222K	15. 222M	16. 222N		

www.everythingmaths.co.za   m.everythingmaths.co.za

Laws of exponents EMBF4

We use exponential notation to show that a number or variable is multiplied by itself a certain number of times. The exponent, also called the index or power, indicates the number of times the multiplication is repeated.

$$\text{base} \longleftarrow a^n \longrightarrow \text{exponent/index}$$

$$a^n = a \times a \times a \times \ldots \times a \quad (n \text{ times}) \qquad (a \in \mathbb{R}, n \in \mathbb{N})$$

See video: 222P at www.everythingmaths.co.za

Examples:

1. $2 \times 2 \times 2 \times 2 = 2^4$
2. $0{,}71 \times 0{,}71 \times 0{,}71 = (0{,}71)^3$
3. $(501)^2 = 501 \times 501$
4. $k^6 = k \times k \times k \times k \times k \times k$

For x^2, we say x is squared and for y^3, we say that y is cubed. In the last example we have k^6; we say that k is raised to the sixth power.

We also have the following definitions for exponents. It is important to remember that we always write the final answer with a positive exponent.

- $a^0 = 1$ ($a \neq 0$ because 0^0 is undefined)
- $a^{-n} = \frac{1}{a^n}$ ($a \neq 0$ because $\frac{1}{0}$ is undefined)

Examples:

1. $5^{-2} = \dfrac{1}{5^2} = \dfrac{1}{25}$
2. $(-36)^0 x = (1)x = x$
3. $\dfrac{7p^{-1}}{q^3 t^{-2}} = \dfrac{7t^2}{pq^3}$

We use the following laws for working with exponents:

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(ab)^n = a^n b^n$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
- $(a^m)^n = a^{mn}$

where $a > 0$, $b > 0$ and $m, n \in \mathbb{Z}$.

Worked example 1: Laws of exponents

QUESTION

Simplify the following:

1. $5(m^{2t})^p \times 2(m^{3p})^t$
2. $\dfrac{8k^3x^2}{(xk)^2}$

3. $\dfrac{2^2 \times 3 \times 7^4}{(7 \times 2)^4}$

4. $3(3^b)^a$

SOLUTION

1. $5(m^{2t})^p \times 2(m^{3p})^t = 10m^{2pt+3pt} = 10m^{5pt}$
2. $\dfrac{8k^3x^2}{(xk)^2} = \dfrac{8k^3x^2}{x^2k^2} = 8k^{(3-2)}x^{(2-2)} = 8k^1x^0 = 8k$
3. $\dfrac{2^2 \times 3 \times 7^4}{(7 \times 2)^4} = \dfrac{2^2 \times 3 \times 7^4}{7^4 \times 2^4} = 2^{(2-4)} \times 3 \times 7^{(4-4)} = 2^{-2} \times 3 = \frac{3}{4}$
4. $3(3^b)^a = 3 \times 3^{ab} = 3^{ab+1}$

Worked example 2: Laws of exponents

QUESTION

Simplify: $\dfrac{3^m - 3^{m+1}}{4 \times 3^m - 3^m}$

SOLUTION

Step 1: Simplify to a form that can be factorised

$$\frac{3^m - 3^{m+1}}{4 \times 3^m - 3^m} = \frac{3^m - (3^m \times 3)}{4 \times 3^m - 3^m}$$

Step 2: Take out a common factor

$$= \frac{3^m(1-3)}{3^m(4-1)}$$

Step 3: Cancel the common factor and simplify

$$\begin{aligned} &= \frac{1-3}{4-1} \\ &= -\frac{2}{3} \end{aligned}$$

Exercise 1 – 2: Laws of exponents

Simplify the following:

1. $4 \times 4^{2a} \times 4^2 \times 4^a$
2. $\dfrac{3^2}{2^{-3}}$
3. $(3p^5)^2$
4. $\dfrac{k^2 k^{3x-4}}{k^x}$
5. $(5^{z-1})^2 + 5^z$
6. $(\frac{1}{4})^0$
7. $(x^2)^5$
8. $\left(\frac{a}{b}\right)^{-2}$
9. $(m+n)^{-1}$
10. $2(p^t)^s$
11. $\dfrac{1}{\left(\frac{1}{a}\right)^{-1}}$
12. $\frac{k^0}{k^{-1}}$
13. $\dfrac{-2}{-2^{-a}}$
14. $\dfrac{-h}{(-h)^{-3}}$
15. $\left(\dfrac{a^2b^3}{c^3d}\right)^2$
16. $10^7(7^0) \times 10^{-6}(-6)^0 - 6$
17. $m^3n^2 \div nm^2 \times \frac{mn}{2}$
18. $(2^{-2} - 5^{-1})^{-2}$
19. $(y^2)^{-3} \div \left(\frac{x^2}{y^3}\right)^{-1}$
20. $\dfrac{2^{c-5}}{2^{c-8}}$
21. $\dfrac{2^{9a} \times 4^{6a} \times 2^2}{8^{5a}}$
22. $\dfrac{20t^5p^{10}}{10t^4p^9}$
23. $\left(\dfrac{9q^{-2s}}{q^{-3s}y^{-4a-1}}\right)^2$

Think you got it? Get this answer and more practice on our Intelligent Practice Service

1. 222R	2. 222S	3. 222T	4. 222V	5. 222W	6. 222X
7. 222Y	8. 222Z	9. 2232	10. 2233	11. 2234	12. 2235
13. 2236	14. 2237	15. 2238	16. 2239	17. 223B	18. 223C
19. 223D	20. 223F	21. 223G	22. 223H	23. 223J	

www.everythingmaths.co.za **m.everythingmaths.co.za**

See video: 222Q at www.everythingmaths.co.za

1.2 Rational exponents and surds EMBF5

The laws of exponents can also be extended to include the rational numbers. A rational number is any number that can be written as a fraction with an integer in the numerator and in the denominator. We also have the following definitions for working with rational exponents.